① If \$8,000 is invested at a rate of 5.5% APR compounded monthly, how long will it take to reach \$20,000?

$$y = \underbrace{8000}_{\text{start}}\left(1 + \underbrace{\frac{0.055}{12}}_{\text{growth rate}}\right)^x$$

$$20000 = 8000\left(1 + \frac{0.055}{12}\right)^x$$

$$2.5 = \left(1 + \frac{0.055}{12}\right)^x$$

② Write an equation for the table and find f(32).

x	1	2	3	4
y	2	3	4.5	6.75

~~0.3152~~

basic exponential function $y = a^x$

$a > 0$

$a \neq 1$

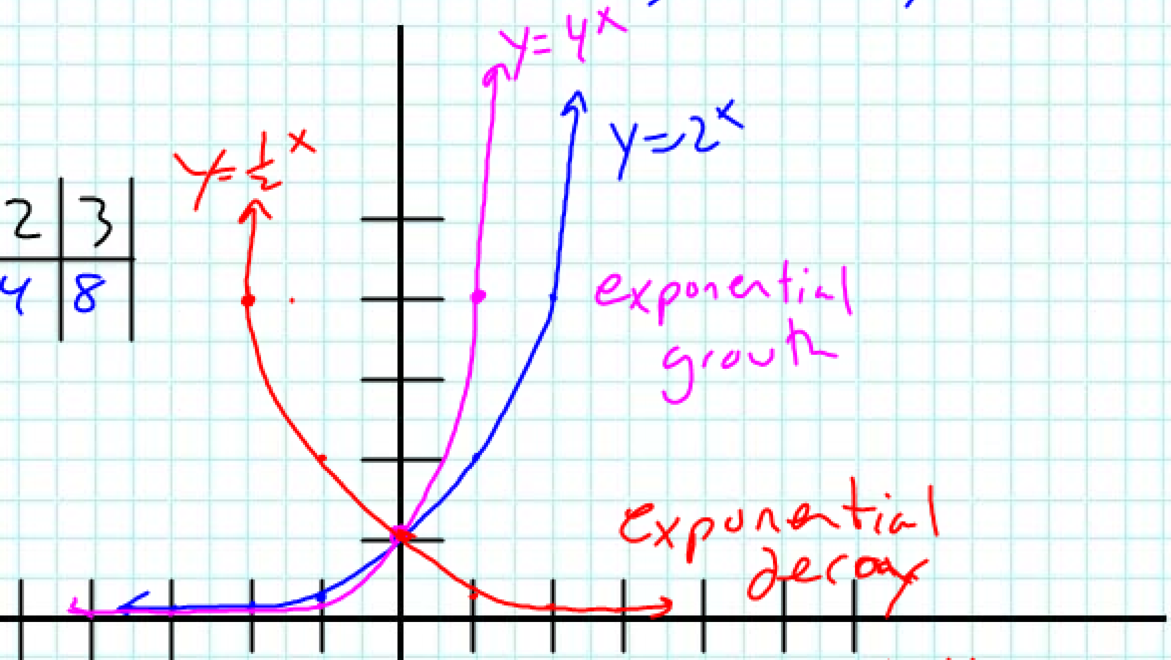
Graph $y = 2^x$, 4^x, $\frac{1}{2}^x$, 2^{-x}

Exponential Function

$y = ab^x, \quad a \neq 0, \; b > 0, \; b \neq 1$

Graph $y = 2^x$

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



Graph $y = 4^x$

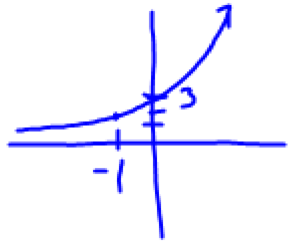
x	-3	-2	-1	0	1	2	3
y	$\frac{1}{64}$	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64

Graph $y = \left(\frac{1}{2}\right)^x$ $\quad y = 2^{-x}$

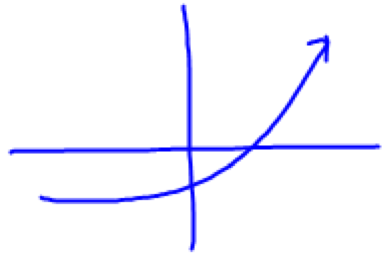
x	-3	-2	-1	0	1	2	3
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

If $f(x) = 3^x$, sketch   NO CALC.

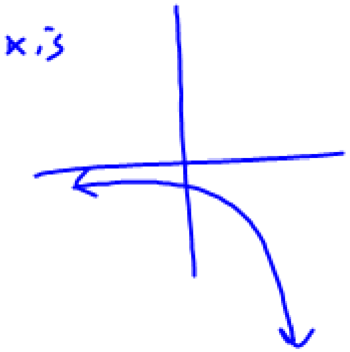
(a) $g(x) = 3^{x+1} \rightarrow f(x+1)$ lft 1



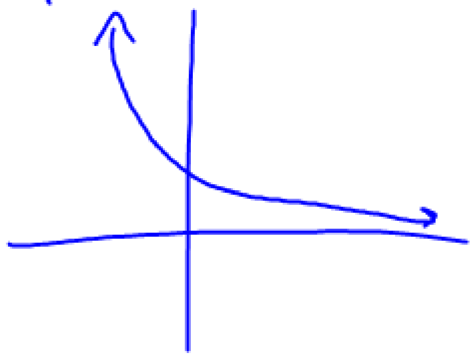
(b) $g(x) = 3^x - 2$ $f(x) - 2$ down 2

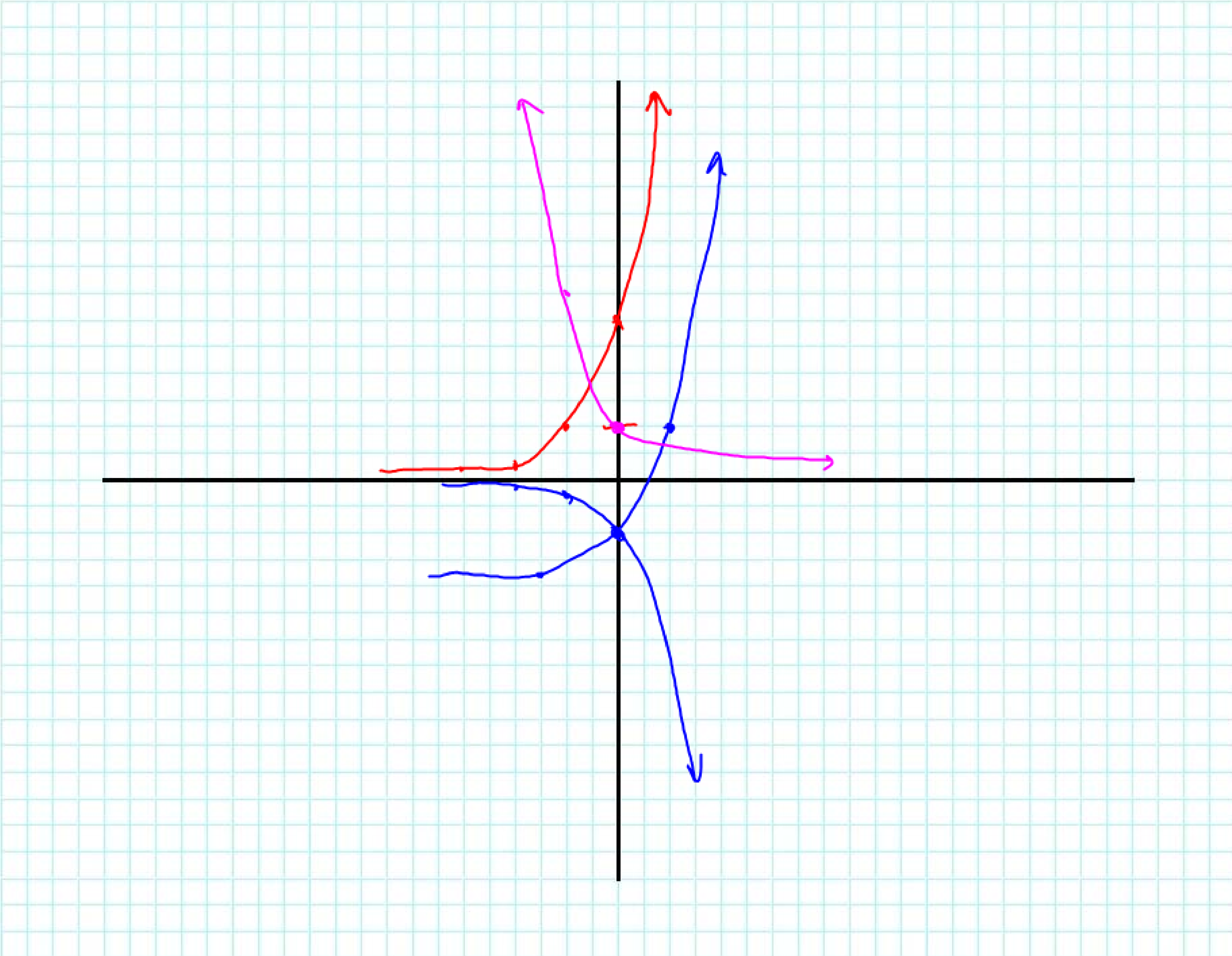


(c) $g(x) = -3^x$ $-f(x)$ flip over x-axis



(d) $g(x) = 3^{-x}$ $f(-x)$ flip over y-axis





Growth factor $b = 1 + r$, $r =$ rate of increase

$y = ab^x$

a → start value

x → often time

Exponential Decay $b < 1$

$b = (1 - \text{rate of decrease})$

6% decrease

$1000(1 - 0.06)^x$

$1000(0.94)^x$

$\cancel{1000(0.06)^x}$

Investments

- APR → Annual percentage rate %

- compounding

  monthly $\frac{APR}{12}$, your x is now months

$8000\left(1+\frac{0.05}{12}\right)^{12x}$ x-yrs

Invest \$5000 at 4% APR compounded monthly

$y = a b^{x}$, with a → start and b → $1+r$

$y = 5000\left(1+\frac{0.04}{12}\right)^{x}$ x → months

$A = P\left(1+\frac{r}{n}\right)^{nt}$

A → Amount at time t

P → initial

r = rate

n = number compounding periods

You invest \$1.00 at 100% APR for 1 year. What is your balance at the end of the year if you compound the interest Keep lots of decimal places

(a) Yearly? 2

(b) Quarterly? 2.4414

(c) Monthly? 2.613035

(d) Weekly? 2.692661

(e) Daily? 2.714567

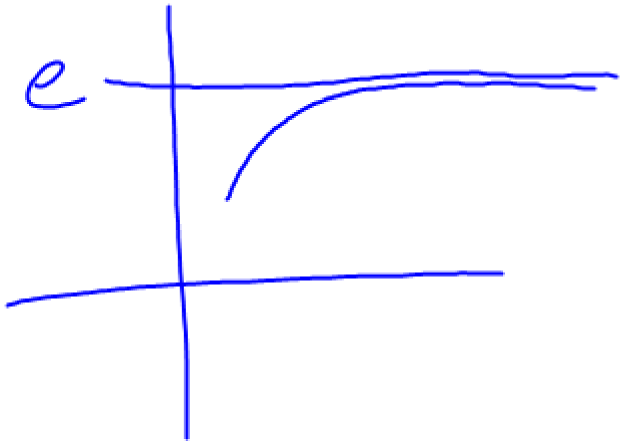
(f) Every hour? 2.7181

(g) Every minute? 2.71827

2.71828

sec.

$\frac{1}{525600}$



$$e = \lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$

e - natural base

$$A = P(1+r)^t$$

$$y = ae^{bt}$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad m = \frac{n}{r} \qquad \frac{1}{m} = \frac{r}{n}$$

$$n = mr$$

$$A = P\left(1 + \frac{1}{m}\right)^{mrt}$$

$$A = P\left(\underline{\left(1 + \frac{1}{m}\right)^m}\right)^{rt}$$

$$A = 1e^{1 \cdot 1}$$

$$= e$$

rate as decimal

$$A = Pe^{rt}$$

time (yrs.)

balance

initial

compounding continuously

Half-Life

You have $75,000 in a retirement account.

Your account loses half its value every 5 years.

(a) Write a model for this situation.

$$y = ab^x$$

$$75000(1-0.5)^{\frac{x}{5}}$$ – every 5 yr's

(b) Find the value after 9 years.

21,538

Continuous Compounding

To compound continuously, we use the natural base (e). We most often use this with natural events of growth (e.g. population) or decay.

$$y = ae^{bx}$$

or as we did in class for an investment

$$A = Pe^{rt}$$

- A → ending amount
- P → start value (principal)
- e → base e
- r → the rate
- t → usually years

Ex. \$3,000 invested at 4.5% APR compounded continuously.

$$A = 3000e^{(0.045 \cdot t)}$$

For 10 years later: $A = 3000e^{(0.045 \cdot 10)}$

$$\boxed{\approx \$4,704.94}$$

Writing an exponential equation through two points

$y = ab^x$ (2, 2) (3, 4)

① $2 = ab^2$

② $\frac{2}{b^2} = \frac{ab^2}{b^2}$

$\frac{2}{b^2} = a$

③ $y = ab^x$

$4 = \frac{2}{b^2} \cdot b^3$

$4 = \frac{2b^3}{b^2}$

$\frac{4}{2} = \frac{2b}{2}$

$2 = b$

④ $\frac{2}{b^2} = a \rightarrow \frac{2}{2^2} = a$

$a = \frac{1}{2}$

Steps

① Start with $y = ab^x$, plug in first pt. for x + y

② Solve for a

③ Plug in a, and 2nd pt. $y = ab^x$ solve for b

④ Sub. your b back into the a equation to find a

⑤ Write equation using a + b

$$\boxed{y = \frac{1}{2}(2)^x}$$

The half-life of a radioactive substance is the time it takes for half of the material to decay. A hospital prepares a 100mg supply of technetium-99m, which has a half-life of 6 hours. Write an exponential function for the amount of technetium-99m after x hours and then find the amount remaining after 75 hours.

$$100\left(\frac{1}{2}\right)^{\frac{x}{6}}$$

$x \rightarrow$ hours

Sect. 3.1

1-6 (1)

7-13 (odd)

15-18

23-28 (1)

40, 47, 48

55-58 (1)

63-70 (2)

Write an equation, $y = ab^x$, through $(2,4)(3,16)$